United We Stand, Divided We Fall: Strategic Supplier Alliances under Default Risk

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We study the alliance formation strategy among suppliers in a one downstream firm-n upstream suppliers framework. Each supplier faces an exogenous default risk and is endowed with certain amount of resources that can mitigate that risk. They can share the resources within an alliance, but need to allocate profits of a surviving alliance among the partners proportional to their resource shares. In this context, suppliers need to decide whether to join larger alliances that have better chances of survival or smaller ones that may grant them higher profit allocations. We first analytically characterize the exact coalition-proof Nash-stable alliance structures that would arise in equilibrium for symmetric complementary and substitutable suppliers. Our analysis reveals that risk-reduction, rather than competition-reduction, is the motivator for alliance formation. In general, a more risky and/or a less fragmented supply base favors larger alliances, whereas substitutable suppliers and customer demands with lower pass-through rates result in smaller alliances. We then characterize the alliance structures for an asymmetric supplier base. This helps us establish that larger alliances are stable when the supplier base is more homogenous in terms of their risk levels, rather than divided among few highly risky suppliers and other low-risk ones. Going one step further, our investigation of endogenous risk levels for the suppliers demonstrates how investment and production costs affect suppliers’ investment in risk-reducing resources, and consequently their alliance formation strategy. Lastly, we discuss model generalizations in terms of profit allocation rule and price premium to be paid by the upstream party in case of alliance default, and show that, in general, our insights are quite robust.

Key words: Cooperation, Competition, Supply risk, Complementary products, Competitive markets

1. Introduction
Strategic alliances, whereby independent but cooperating organizations pool specific resources and skills in order to achieve common and individual goals, have emerged as a popular strategy in the business world (Varadarajan and Cunningham 1995, Gulati 1998). While such alliances can be between horizontal and/or vertical partners, throughout this paper we focus on horizontal
alliances. Examples of these can be observed between firms producing complementary products as well as between competitors selling substitutable products. There are a number of complementary-products alliances; consider the ones between Caterpillar and Mitsubishi in the earthmoving equipment sector, and among component suppliers in automobile and electronics sectors (Nagarajan and Sošić 2009). On the other hand, examples of substitutable-product alliances include Renault and Nissan in the automobile industry (Yoshino and Fagan 2003), Takeda and Hoechst in the pharmaceutical sector (Garella and Peitz 2007), and collaborative organizations in petrochemical (Khashagir and Roy 2009) and agricultural sectors (Oxfam International 2010).

There is considerable amount of literature, especially in the strategy and organization area, analyzing alliances from multiple perspectives. For example, they deal with governance structures of alliances (Gulati and Singh 2008) as well as effects of alliances on firm performance (Singh and Mitchell 2005), innovation rate (Stuart 2000), knowledge transfer (Mowery et al. 1996), market access (Varadarajan and Cunningham 1995) and bargaining power (Hamel 1991). This literature suggests that one of the most important reasons behind alliance formation is to deal with external business risks (e.g., Khanna and Yafeh 2005 and Mitchell and Singh 1996). Since supply chains face both supply and demand risks, this risk-mitigating role is particularly relevant. Indeed, a number of papers in the operations management literature have studied alliance formation in the presence of demand-side risk (refer to Nagarajan and Sošić 2009 and §2 for detailed reviews).

However, in recent times, the growth in outsourcing has brought default risk of the suppliers to the forefront. Specifically, as supply chains become longer and more decentralized, factors such as political uncertainty, weather, financial crisis, terrorism and strikes make a total breakdown of the supply system more likely. Examples include disruption in oil supply from some countries due to political instability (Reuters 2007), frost wiping out most of California’s citrus crops in 1998 (Rimal and Schmitz 1999), the recent earthquake and tsunami disrupting supply from Japan (NYTimes 2011), and default of more than 10,000 factories in China during the financial crisis of 2008 (USA Today 2008). A number of strategies have been proposed to deal with supply risks including diversification (Babich et al. 2007), subsidies (Wang et al. 2010, Babich 2010, Wadecki et al. 2012), guarantees (Gümüs et al. 2012), and contracting (Swinney and Netessine 2009, Yang et al. 2009). However, if we take the agriculture industry as an example, then one strategy used by the producers is to form alliances so as to deal with default risks arising from factors such as bad weather, transport losses or difficulty in accessing capital. Such alliances, e.g., Farmer Producer Organizations (FPOs) in India, which use supply chain risk-management funds and

1 Note that we will use the terms “alliance” and “coalition” interchangeably throughout the paper.
2 Obviously, such alliances also provide benefits like better access to markets and more negotiation power.
shared investments to improve the functioning of the chain, are becoming popular all over the world\(^3\). But, alliances are aware that such partnerships require equitable sharing of risks and benefits, and consequently are not easy to sustain (Oxfam International 2010, Government of India 2013). Interestingly, analytical investigation as to what types of stable alliances\(^4\) will develop in presence of supply risk is sparse in the academic literature. This paper attempts to address this gap.

We consider a bi-level supply chain model framework composed of \(n\) upstream suppliers and one downstream firm (henceforth referred to by masculine and feminine pronouns, respectively). The downstream firm faces a price-sensitive and deterministic demand that she needs to satisfy by procuring the required components - complements or substitutes - from the suppliers. Each supplier faces a default risk and is endowed with a certain amount of exogenous resources. These resources can *mitigate* default risk, but the risk-mitigating benefit shows *diminishing marginal returns*. Moreover, the resources are generic and liquid enough (e.g, cash, knowledge) so that they can be *shared* with (or transferred to) other suppliers, irrespective of whether they are complements or substitutes. Note that resource endowment and default risk are inversely related - the higher is the amount of resources available to a supplier, the lower is his effective default risk\(^5\).

The suppliers first decide on their *cooperative* alliance structure by determining whether to join an alliance, and, if so, with how many other partners. Since the resources are shareable, the alliance partners can pool their resources to lower the risk of the alliance. Each alliance then announces its supply limit for the downstream firm and *competes* horizontally via wholesale prices. Subsequently, the downstream firm decides on the order quantity for each alliance and on the retail price. Finally, default risks and consequent profits are realized for the alliances. Each alliance then divides its profit among the partners following a pre-determined allocation rule that is proportional to the resource share of each partner. We use the above framework to address the following issues:

- What is the equilibrium stable alliance configuration that would arise under the risk of default?
- What factors incentivize the suppliers to opt for larger (or smaller) alliances?
- How robust are these results with respect to model assumptions? For example:
  - What if the resource endowment is an endogenous decision?
  - What if the profit allocation is not proportional to the resource shares?

We first focus on the case of suppliers who are symmetric in terms of their risk levels and analytically characterize the number of stable alliances and their sizes through coalition-proof

\(^3\) Note that these new alliances are for-profit in nature, rather than not-for-profit cooperatives. Also, many believe that a prime reason behind Renault-Nissan alliance was the potential for default on the part of Nissan (Singh 2013).

\(^4\) An alliance is stable if there is no incentive for alliance partners to profitably deviate from it.

\(^5\) In this paper we will use resource and default risk interchangeably, keeping in mind their inverse relationship.
Nash equilibrium technique. Note that the suppliers need to trade-off the benefits of joining larger alliances that grant them higher survival probability against the benefits of greater profit allocation in smaller alliances while making alliance decisions. Alliance literature in supply chain management area until now focused primarily on the second factor and hence suppliers usually end up forming small alliances (see Nagarajan and Sošić 2007 and Yin 2010 for discussions). In contrast, by incorporating supply side risks, we are able to identify a diverse set of stable alliance structures, both large and small, depending on the business environment.

Specifically, we identify a novel risk-adjusted stability factor, which encapsulates the characteristics of the supply base and customer demand, to determine the stable alliances. Analysis of this factor shows that larger alliances (including a grand coalition of all \( n \) suppliers) are more likely to be formed when: i) the supplier base is more risky and/or relatively small, ii) suppliers are complementary, and iii) retail price is more sensitive to wholesale prices (higher pass through rates). On the other hand, antithetical business conditions (e.g., less risky suppliers or lower pass through rate) result in smaller alliances and might even incentivize suppliers to work alone. Interestingly, it is risk-reduction through resource sharing that provides the incentive for alliance formation, rather than competition reduction through collaborative decision-making.

Subsequently, we generalize the above model to account for an asymmetric supplier base consisting of a certain number of more risky suppliers and some less risky ones, and once again analytically characterize the sizes of stable alliances. This characterization requires a modification of the stability factor to account for the asymmetry. It turns out that our previous insights remain valid; in addition, we show that a more homogenous (resp., heterogenous) supplier base results in larger (resp., smaller) alliances. We then go one step further by making resource endowment decisions endogenous, i.e., suppliers decide their alliance partners as well as how much they want to invest in risk-reducing resources. If investment is costly, suppliers like to take advantage of risk reduction through resource sharing by forming large alliances, whereas if the investment is cheap, they would like to go alone in order to have a higher profit allocation. We also comment on the case where the investment in supply-risk-reducing resources is made by the downstream firm and illustrate how this decision is impacted by whether the firm is an assembler or a buyer (i.e., dealing with complementary or substitutable suppliers, respectively).

Lastly, we test the robustness of the above qualitative insights through two generalizations of our modeling framework: i) profit allocation mechanism that is not proportional to resource shares, and ii) a non-trivial default premium that needs to be paid by the downstream firm in case of a supplier default. The main insight of the first generalization is that larger alliances are sustained for relatively fair allocations (either if they are naturally fair or if they are induced to be fair by providing favoritism towards minority supplier types). As regards to the second generalization, all
our previous insights hold as long as the premium is not too high. As one would expect, a higher default premium increases the sizes of the alliances.

The rest of the paper is organized as follows. The next section discusses the related literature, while §3 presents our basic modeling framework with exogenous resource endowments. The operational decisions of this model are analyzed in §4, while §5 deals with alliance formation decisions for both symmetric and asymmetric supplier bases. §6 studies endogenous resource endowment scenario while §7 concerns two model generalizations. The concluding discussion is provided in §8.

2. Literature Review

There are two streams of literature most directly related to our work: research dealing with alliance formation but where supply side risk (or resource availability to reduce such risk) is not considered, and research studying measures to counteract supply default risk, but where suppliers only compete with each other (i.e., without any coalition/alliance consideration).

Our modeling framework of multiple-suppliers-one-downstream-firm channel has a long history in operations literature. Papers in this area traditionally had a competitive focus, e.g., Wang and Gerchak (2003), Jiang and Wang (2010) for complementary suppliers, Bernstein and Federgruen (2005), Yang et al. (2012) for substitutable suppliers, and Netessine and Zhang (2005) for both types of suppliers, to name a few. In these papers, suppliers make individual decisions that maximize their own profits taking into account the responses of competing firms. The possibility for suppliers to communicate and jointly set their prices and/or production quantities, supply limits and such, is not considered.

In recent years, a line of research studying the coalition structures that could arise among collaborating suppliers has emerged. For example, Nagarajan and Sošić (2007) investigate the stability of alliances among suppliers selling substitutable products in a dynamic setting. Suppliers are assumed to be farsighted and take into account possible future defections when making any immediate decision. On the other hand, Nagarajan and Bassok (2008) study alliance stability among complementary suppliers when they can negotiate with the downstream assembler about profit allocations. They find that grand coalition (resp., no coalition) will emerge if the bargaining power of the assembler is weak (resp., strong). Also in the context of assembly systems, Granot and Yin (2008) find that coalitions are more likely to be formed in a pull system than a push one, and in the latter case whether the suppliers will form a grand coalition or act independently depend on their perspective about cooperation (farsighted or myopic). Nagarajan and Sošić (2009) consider three modes of competition among complementary suppliers, and analyze stable coalitions as a function of power structure, demand structure, and the number of suppliers. Under a similar framework, Sošić (2011) studies the impact of demand uncertainty on the alliance structures. Lastly,
for a quite general market condition, Yin (2010) explicitly characterizes stable coalition structures in assembly systems, and their dependence on demand conditions.

Note that all of the above papers deal only with demand-side risks. In general, the incentive for alliance formation in this literature has been attributed to the channel/market structure (Granot and Yin 2008, Nagarajan and Sošić 2009, Yin 2010), bargaining power (Nagarajan and Bassok 2008), the cooperative perspective of the players (Nagarajan and Sošić 2007, Granot and Yin 2008, Nagarajan and Sošić 2009) and the nature/extent of demand uncertainty (Yin 2010, Sošić 2011). We follow this literature by also investigating how the stability of supplier alliances is affected by a variety of business conditions, but complement it by showing that the possibility of default risk itself can also be a significant incentive behind alliance formation.

As regards the second stream, among the vast literature related to exogenous supply risks, our paper particularly emphasizes supply disruption/default risk, because of which a buyer may lose all its order (refer to Kleindorfer and Saad 2013 and Sodhi et al. 2011 for reviews about supply risk in general). Previous studies have taken a wide angle regarding this issue. Analyzing from the buyer’s perspective, Tomlin (2006) considers several mitigation measures and contingency tools to hedge against a variety of disruptions. Babich et al. (2007) and Chopra et al. (2007) investigate the impact of risk correlation and the type of risk (recurring or disruption) facing the supplier community on optimal sourcing diversification decisions, respectively. Swinney and Netessine (2009) analyze the value of long-term vs. short-term contracts in the presence of a default risk, and Yang et al. (2009) derive the optimal contract when suppliers hold private information about their reliability. Chaturvedi and Martínez-de Albéniz (2011) extend Yang et al. (2009) by also including supplier’s cost as private information. Lastly, Saghafian and Van Oyen (2012) show the value of having a flexible backup supplier in the presence of disruption risk and discuss capacity reservation issue in that context. Looking from the suppliers’ perspective, Gümüş et al. (2012) study the impact of guarantees on risk mitigation and the ability of suppliers to signal their true risk levels. Wei et al. (2013) discuss the implications of default risk coming from uncertain market prices or valuations, when the buyer can use vertical subsidy as a strategic measure. Note that there is a vast operations management literature exploring the impact of yield risk on operational decisions. Among the recent ones, Tomlin (2009), Kazaz and Webster (2011) and Gurnani et al. (2012) investigate the effects of learning, yield-dependent cost structure, and information asymmetry on operational/marketing decisions, respectively. In general, the papers in this research stream have had either a centralized or a competitive focus. To the best of our knowledge, the current paper is among the first that considers disruption/default risk in a cooperative context, and is able to establish its role in suppliers’ alliance formation decision.
Note that, although our focus is on horizontal collaboration, there is a rich stream of literature dealing with vertical collaboration (refer to Paulraj et al. 2008, Kim and Netessine 2013). There are also two other growing streams that are in spirit related to our work: i) Empirical analysis of the causes of horizontal alliance formation (e.g., Li and Netessine 2011), and ii) horizontal mergers in supply chains (e.g., Cho 2013). However, these streams differ from this paper in terms of methodology as well as model setting.

3. Model Framework
Consider a supply chain with a single downstream firm procuring components from \( n \) upstream suppliers and selling a final product to end consumers. In our setting, the components can either be complements or substitutes. In case they are complements, the final product is an assembly consisting of one component each from \( n \) suppliers. On the other hand, if the components are substitutes, the final product consists of only one component, which is available from any of the \( n \) suppliers. The downstream firm - an assembler or a buyer depending on whether the components are complements or substitutes, respectively - acquires the required components from the suppliers and then (costlessly) assembles/produces the end product to satisfy customer demand (refer to Figure 1). Below, we describe the salient features of the stakeholders in our supply chain.

![Figure 1 Channel Structures with Complementary and Substitutable Suppliers](image)

**Downstream firm:** We model the end product demand facing the downstream firm as a price-sensitive deterministic function \( D(p) \) where \( p \) is the price set by her. We assume that \( D(p) \) is positive and decreasing in \( p \), and its price elasticity satisfies the following form

\[
\eta(p) = -\frac{D'(p)}{D(p)} \frac{p}{p} = \frac{p}{\alpha + \beta p}.
\]

where \( \alpha \geq 0 \) and \( \beta \leq 1 \) (as in Song et al. (2007)). Note that the class of demand functions that satisfy (1) is quite general and subsumes most of the specific demand forms assumed in the related
literature, such as iso-elastic (Wang 2006), linear (Nagarajan and Sošić 2007, Nagarajan and Sošić 2009) as well as linear-power and exponential (Yin 2010); see Table A2 in the Appendix for details.

Upstream suppliers: Each of the $n$ upstream suppliers faces a default risk in terms of order delivery. Following the recent supply risk literature (e.g., Babich et al. 2007, Yang et al. 2009, Gümüş et al. 2012) we assume that these risks are independent of each other and of an all-or-nothing kind. The last feature implies that with a certain probability each supplier can deliver either the whole order or nothing. Moreover, each supplier is endowed with a certain amount of exogenous resources to mitigate his default risk.\(^6\) Rather than explicitly modeling the details of risk-reduction process, in line with the parsimonious approach adopted in related literature (e.g., Shavell 1984, Wadecki et al. 2012, and references therein) we also assume the resources to be some form of capital or knowledge (e.g., cash flow, liquid assets, process or technology knowledge) that reduces default risk. The resource is not relation- or product-specific; rather it is generic and liquid enough to be shared with other complementary or substitute suppliers to reduce their default risks. For example, in the agricultural sector, such shareable resources can be cash, technology like harvesting/transportation equipment and knowledge in the form of cultivation techniques or weather. So, if a supplier forms cooperative alliances with an arbitrary number of other suppliers, it allows them to pool their resources and reduce the default risk of the alliance.\(^7\)

Let $G(F)$ denote the survival probability of a supplier whose resource endowment is $F$. The complement of $G(F)$ is given by $\bar{G}(F) = 1 - G(F)$. We assume $G(F)$ satisfies several regularity conditions. First, it is between 0 and 1. Second, $G(F)$ is an increasing function of $F$, implying that a supplier with a higher amount of resources should have a lower chance of default. Finally, to capture the diminishing returns of risk-mitigating resources on the survival probability, we assume that $G(F)$ is concave. In order to satisfy all these three conditions in a tractable fashion, we take $G(F) = \frac{F}{\lambda + F}$ with $\lambda > 0$. Without loss of generality, we normalize $\lambda$ to 1 and let $G(F) = \frac{F}{1 + F}$.

We operationalize the suppliers’ incentive to form a coalition as follows. Recall that, when suppliers form an alliance\(^8\), they can pool and share their resources. This implies that the resource endowments of the coalition members collectively determine the survival probability of the coalition. To be specific, let $S_k$ be an alliance composed of a subset of the suppliers. Then, the survival probability of the alliance $S_k$ is given by $G(F_{S_k})$, where $F_{S_k} = \sum_{i \in S_k} F_i$ represents the total resource endowment of the alliance.

\(6\) In §§6.1 and 6.2, we extend our base model by allowing parties to decide on how much risk-reducing resources to invest in, thus endogenizing supply risk levels.

\(7\) For a detailed modeling of the default process of the suppliers refer to Swinney et al. (2011). Also, we suggest the modeling of cases where the resources are relation- or product-specific, e.g., whether the suppliers are complements or substitutes, and hence might not necessarily be shareable, as avenues for future research.

\(8\) A supplier that does not join any alliance can be thought of as an alliance of size 1.
Forming an alliance increases the survival probability of its members, but the profit of a surviving alliance needs to be shared among the partners. We assume that each coalition employs a predetermined profit allocation mechanism that depends on each member’s contribution to the total resource endowment. More specifically, we let $\gamma_{i,S_k}$ denote the fraction of profit allocated to supplier $i$ in alliance $S_k$. Then, supplier $i$ gets a profit allocation that is proportional to his endowment $F_i$, i.e.,

$$\gamma_{i,S_k} = \frac{F_i}{\sum_{j\in S_k} F_j}, \forall S_k \text{ and } i \in S_k$$

(2)

The above allocation mechanism is quite intuitive and “fair”. Nevertheless, we discuss certain possible generalizations in §7.

Next, we turn our attention to how the players compete and cooperate in our model. A glossary of notations used in the paper is provided in Table A1 of the appendix. In particular, we use $|S_k|$ to represent the number of suppliers in the set $S_k$.

**Game sequence:** The sequence of the events and decisions is as follows (refer to Figure 2).

*Stage 1:* Upstream suppliers strategically form alliances $\mathcal{S} = \{S_1, S_2, ..., S_m\}$, where $\mathcal{S}$ is an $m$-partition \(^9\) of the supplier set $N$, by playing a cooperative game among themselves.

*Stage 2:* The alliances commit to supply limits $\{Q_{S_i}\}$, which cap the amount each alliance will produce. For technical exposition, we assume zero commitment cost.\(^{10}\) Alliances then determine their wholesale prices $\{w_{S_i}\}$. Note that this stage involves competitive decision-making.

*Stage 3:* The downstream firm maximizes her profit by determining the retail price $p$ for the final product and the order quantities $\{q_{S_i}\}$, $q_{S_i} \leq Q_{S_i}$, for each alliance.

*Stage 4:* Default risk resolves. If an alliance $S_k$ survives, the entire order from the downstream firm will be delivered. Each unit produced by alliance $S_k$ will incur a marginal cost $c_{S_k}$, where $c_{S_k}$ is equal to $|S_k|c$, and $c$ for the complementary and substitutable cases, respectively. In addition, the resulting profit is shared among the alliance members based on the allocation rule (2). Otherwise, in case of default, the alliance receives no payment, and the downstream firm has to utilize an emergency source, where we assume that the emergency source charges the firm a premium $\delta_{S_k}$ on top of the alliance’s wholesale price $w_{S_k}$. When the components are complementary, there is a unit premium $\delta \geq 0$ for each component, and therefore $\delta_{S_k} = |S_k|\delta$; if they are substitutes, there is only one component from the alliance, and therefore $\delta_{S_k} = \delta$. Subsequently, the downstream firm sells the final product to the end customers at price $p$ and collects her revenue.\(^{11}\)

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\(^9\) Given a set of suppliers $N$, the structures $\mathcal{S} = \{S_1, ..., S_m\}$ is an $m$-partition of $N$ if it satisfies $\bigcup_{k=1}^{m} S_k = N$ and $S_i \cap S_j = \emptyset$ for any $1 \leq i < j \leq m$.

\(^{10}\) This assumption is without loss of generality. More discussion can be found in the appendix.

\(^{11}\) Premium emergency sourcing option has been used before in the related literature, e.g., Dong and Tomlin (2012).
4. Operational Decisions under a Given Alliance Structure

In this section, using backward induction, we characterize the equilibrium operational decisions of each alliance and the downstream firm (i.e., stages 2 and 3 of the game) given an alliance structure \( \mathcal{I} = \{S_1, \ldots, S_m\} \).

We start with optimal ordering and pricing decisions of the downstream firm (i.e., stage 3). To make her ordering and pricing decisions, the downstream firm will take into account the possibility that each alliance \( S_k \) might default with certain probability, in which case she would have to pay a premium \( \delta_{S_k} \) to ensure supply as discussed in the last section. This allows us to determine the expected wholesale price paid by the downstream firm to each alliance for both complementary and substitutable cases. Based on the expected wholesale prices, the downstream firm then determines the expected-profit-maximizing order quantities from the alliances (which determine the retail price for the final product based on the inverse demand function). The equilibrium decisions are given in Proposition 1. All the technical proofs are provided in the Appendix.

**Proposition 1. (Equilibrium Operational Decisions)** Given an alliance structure \( \mathcal{I} = \{S_1, \ldots, S_m\} \), the equilibrium decisions for stages 2-3 are as shown in Table 1.

The equilibrium solutions inherit a structure similar to those without default risk (e.g., Yin 2010 for the complementary case) and adjust it to account for the risk-reducing resources \( F_{S_k} \) of the alliances, i.e., their effective supply risks. Indeed, \( F_{S_k} \) significantly affects the decisions for both complementary and substitutable suppliers, albeit somewhat differently. For example, in the complementary case, the equilibrium alliance production quantity \( q^*_{S_k} = Q^*_{S_k} \) is affected by the effective total production cost \( \bar{C} \) reflecting the aggregated effect of all endowment levels \( \{F_{S_i}\}_{i=1}^m \). Thus, higher resource endowments result in higher order quantities for every complementary alliance (\( \bar{C} \) and \( q^*_{S_k} \) are inversely related). But under the substitutable case, \( q^*_{S_k} \) is affected via both effective
Table 1  Equilibrium Operational Decisions for Stages 2 and 3

<table>
<thead>
<tr>
<th></th>
<th>Complementary suppliers</th>
<th>Substitutable suppliers</th>
</tr>
</thead>
<tbody>
<tr>
<td>Retail Price ($p^*$)</td>
<td>$\frac{m\alpha + \bar{C}}{(1-m\beta)(1-\beta)} + \frac{\alpha}{1-\beta}$</td>
<td>$\frac{\alpha + m\bar{c}}{(m-\beta)(1-\beta)} + \frac{\alpha}{1-\beta}$</td>
</tr>
<tr>
<td>Wholesale Price ($w_{S_k}^*$)</td>
<td>$\frac{\alpha + \beta\bar{C}}{1-m\beta} + c_{S_k}$</td>
<td>$\frac{\alpha + \beta\bar{c}}{m-\beta} + c + \tilde{c} - \tilde{c}_{S_k}$</td>
</tr>
<tr>
<td>Order Quantity ($q_{S_k}^*$)</td>
<td>$D(p^*)$</td>
<td>$D(p^*) \left(1 + \frac{m - \beta (\tilde{c} - \tilde{c}_{S_k})}{\alpha + \beta\bar{c}} m\right)$</td>
</tr>
</tbody>
</table>

where $\bar{C} = nc + \sum_{i=1}^{m} \bar{G}(F_{S_i})\delta_{S_i}$, $\bar{c} = c + \sum_{i=1}^{m} \bar{G}(F_{S_i})\delta / m$, $\tilde{c}_{S_k} = c + \bar{G}(F_{S_k})\delta$ and $\bar{G}(F_{S_k}) = 1 - G(F_{S_k})$.

average production cost $\bar{c}$ (another aggregated representation of $\{F_{S_i}\}_{i=1}^{m}$) and effective individual production cost $\tilde{c}_{S_k}$ (related to $F_{S_k}$ only), which determine the total order quantity and the order allocation among alliances. In this case, although higher resources still result in total quantity expansion, alliance $S_k$’s actual order also depends on his own resource level compared to his competitors. The sensitivity of the profits with respect to alliance structures are also quite different for upstream and downstream firms, as shown in the following corollary.

**Corollary 1. (Preferred Alliance Structures)** As the number of alliances $m$ increases (i.e., suppliers cooperate less),

(i) when the suppliers are complementary, the ex-post profit of each alliance decreases, the expected profit of the downstream firm (assembler) decreases, and consumer surplus decreases;

(ii) when the suppliers are substitutable, the ex-post profit of each alliance decreases, the expected profit of the downstream firm (buyer) increases, and consumer surplus increases.

The essence of Corollary 1 is that, while upstream suppliers are always better off with larger alliances (small $m$), the same may not be true for the downstream firm. In particular, a downstream firm that deals with complementary suppliers would generally prefer larger alliances (e.g., highly integrated sub-assemblies) because they indirectly reduce the cost of default risk for the suppliers as well as the intensity of indirect competition\(^{12}\), which in turn enables them to sustain lower equilibrium wholesale prices. On the other hand, a downstream firm that deals with substitutable suppliers would prefer smaller alliances (e.g., highly fragmented market) in order to strengthen the competition, which would in turn depress the equilibrium wholesale prices. Obviously, lower wholesale prices result in lower retail prices to improve consumer surplus (and vice versa).

\(^{12}\) See Jiang and Wang (2010) for an explicit explanation of indirect competition among complementary suppliers.
5. Alliance Structure and Stability

Before presenting the characterization of stable equilibrium alliance structures among upstream suppliers, we first discuss the stability concept used in this paper.

5.1. Coalition-Proof Nash Equilibrium (CPNE)

A standard stability concept for coalitions in cooperative game theory is Nash equilibrium (NE). An NE coalition structure is defined as the one in which there is no *individual* profitable deviation for any party. Even though NE allows for a simple verification, it does not account for profitable deviations as a *group*. As a remedy for this, the concept of Strong Nash Equilibrium (SNE) has been proposed. A coalition structure is SNE if it is immune to deviation by any arbitrary set of suppliers. However, this concept suffers from imposing too strong conditions on the coalitions and lacks consistency in definition (Bernheim et al. 1987). As an alternative, this paper adopts a refined stability concept, called *coalition-proof Nash equilibrium* (CPNE, refer to Bernheim et al. 1987 for details). In general,

- a CPNE must be *self-enforcing* and not strictly dominated by another *self-enforcing* strategy;
- a strategy among a group of players is *self-enforcing* if every subgroup plays CPNE strategy in its component game.

Therefore, unlike NE, CPNE allows suppliers to communicate and deviate as a group. However, CPNE does not consider *all* potential deviations as in SNE, but only the valid (by definition, *self-enforcing*) deviations — that no proper subset of the defecting players can reach a mutually beneficial agreement to deviate from the deviation. In this sense, CPNE is more consistent and forward-looking than SNE. In particular, it allows explicit characterization of all stable outcomes. Due to these reasons, CPNE has been used in the literature for analyzing alliance formation in a number of settings, including some involving generic risks (Bernheim and Whinston 1987; Genicot and Ray 2003). Throughout the paper, we use “stable” to abbreviate “coalition-proof Nash stable”, unless specified otherwise.

Given a coalition structure \( \mathcal{S} = \{S_1, ..., S_m\} \), the expected profit of a particular supplier \( i \) in an alliance \( S_k \) is given by

\[
\Pi_i(\mathcal{S}) = \pi_{S_k}(\mathcal{S})G(F_{S_k})\gamma_{i,S_k}, \quad \forall i \in S_k.
\]  

(3)

The profit function involves three terms. The first term \( \pi_{S_k}(\mathcal{S}) \) denotes the profit for alliance \( S_k \) if it survives, which can be derived from Proposition 1. The second term, \( G(F_{S_k}) \), measures the survival likelihood of a coalition; it represents the *strategic benefit* of an alliance. So, \( \pi_{S_k}(\mathcal{S})G(F_{S_k}) \)}
is the expected profit of alliance $S_k$ taking into account the default risk. Lastly, $\gamma_{i,S_k}$ (refer to (2)), denotes the share of coalition profit allocated to supplier $i$, reflecting the (dis)benefit of an alliance.

In order to focus on the key trade-off, for now, we assume that $\delta = 0$. That is, the price premium $\delta$ that the downstream firm has to pay in case of supplier default is minimal. So, our basic framework represents a scenario where, if a supplier fails, then there are plenty of external options available to “match” his price.\textsuperscript{13} We will discuss the impact of $\delta > 0$ in $\S$7.2.

5.2. Symmetric Suppliers

In this section we consider identical suppliers facing the same level of default risk — that is, each of them has the same amount of risk-mitigating resources $F$. In this context, we first precisely characterize the stable equilibrium alliance structures and then discuss what factors affect the incentive for alliance formation.

Since all suppliers have the resource level $F$, a coalition $S$ that consists of $|S|$ suppliers would have total resources $F_S = |S|F$ and survival rate $G(F_S) = \frac{|S|F}{1 + |S|F}$. Our first result states that possible stable coalitions should be of similar sizes. In particular, stability disallows any two coalitions differ by more than one supplier. Therefore, if the suppliers form $m$ coalitions, there is only one configuration that is possibly stable, which is the one that has $n - m\lfloor n/m \rfloor$ coalitions of size $\lfloor n/m \rfloor$, and $m + m\lfloor n/m \rfloor - n$ coalitions of size $\lfloor n/m \rfloor$, where $\lfloor n/m \rfloor$ and $\lfloor n/m \rfloor$ denote the nearest integers that are (weakly) smaller and larger than $n/m$, respectively.

**Proposition 2.** Consider $n$ suppliers with identical endowment levels. Then, if CPNE contains $m$ coalitions, the size of each coalition should be equal to either $\lfloor n/m \rfloor$ or $\lceil n/m \rceil$.

**Example 1.** Suppose there are $n = 5$ suppliers. Let $I_k$ represents a set of $k$ identical suppliers. Then, Proposition 2 states that the stable coalition structures among the five suppliers can only be of the following form: \{I_5\}, \{I_2, I_3\}, \{I_1, I_2, I_2\}, \{I_1, I_1, I_1, I_2\}, or \{I_1, I_1, I_1, I_1, I_1\}. $\square$

To determine which one of the above structures will be stable, we need to verify the following conditions characterized by the term $U(m)$, where $U(m) = \frac{\pi(m)}{\pi(m+1)}$ is the ratio of ex-post payoff for $m$-supplier-coalition versus $(m+1)$-supplier-coalition (see Table A4 in the Appendix for $U(m)$ expressions for different customer demand forms).

**Theorem 1.** (CPNE with $n$ identical suppliers) For $n$ suppliers with identical endowment levels $F$, there exists a unique CPNE with $m^*$ coalitions. In particular, the suppliers

(i) will form a grand coalition ($m^* = 1$) if $U(1) \geq \frac{1 + nF}{1 + F}$;

\textsuperscript{13}The applicability of this assumption depends on the nature of the components. The more commoditized are the components, the more likely it is that $\delta = 0$ and vice versa for specialized components. Indeed in many instances, the downstream firm mainly incurs a lump-sum cost due to a supplier default, to cover costs related to activities like expediting, implying that $\delta = 0$ effectively.
(ii) will act independently \((m^* = n)\) if \(U(m) < \frac{1 + \lfloor n/m \rfloor F}{1 + F}\) for any \(1 \leq m \leq n - 1\);

(iii) will form \(m^*\) coalitions where \(1 < m^* < n\), if \(U(m) < \frac{1 + \lfloor n/m \rfloor F}{1 + F}\) for any \(1 \leq m \leq m^* - 1\), and \(U(m^*) \geq \frac{1 + \lfloor n/(m^*) \rfloor F}{1 + F}\).

Theorem 1 suggests an algorithm to find the number of stable coalitions among \(n\) suppliers. More specifically, the number of stable coalitions among \(n\) suppliers is determined by the smallest \(m\) (where \(m\) is between 1 and \(n - 1\)) at which \(U(m)\) exceeds \(\frac{1 + \lfloor n/m \rfloor F}{1 + F}\). Equivalently, we can define for each \(m \in \{1, 2, ..., n - 1\}\) a Risk Adjusted Stability Factor \((RASF_m)\) as follows:

\[
RASF_m = U(m) - \frac{1 + \lfloor n/m \rfloor F}{1 + F}
\]  

(4)

and then characterize the number of stable coalitions by searching for the smallest \(m\) where \(RASF_m\) becomes non-negative. This algorithm can be illustrated via the following example.

Example 2. Suppose that there are \(n = 5\) suppliers, and that the market demand is linear with \(D(p) = 9 - 2p\). This results in, by Table A4, \(U(m) = (1 + \frac{1}{m+1})^2\) for both complementary and substitutable suppliers. We can then plot \(RASF_m\)s as functions of \(F\) as shown in Figure 3.

![Figure 3](image-url)  

**Figure 3**  
Risk adjusted stability factor (RASF) in a five-supplier system with linear demand

Clearly, for low values of \(F\) \((F \leq \frac{5}{11} \approx 0.45)\), \(m = 1\) is the smallest index that makes \(RASF_m\) non-negative. Therefore \(m^* = 1\) and the grand coalition \(\{I_5\}\) is uniquely stable. As \(F\) increases, the sizes of the coalitions decrease. When \(\frac{5}{11} \approx 0.45 < F \leq \frac{7}{11} \approx 0.64\), \(RASF_2\) becomes the first
non-negative RASF, hence $m^* = 2$ (i.e., two stable coalitions $\{I_2, I_3\}$). By similar argument, if $\frac{7}{11} \approx 0.64 < F \leq \frac{9}{7} \approx 1.29$ then three coalitions will be formed and $\{I_1, I_2, I_2\}$ is stable. Finally, when $F$ is very high ($F > \frac{9}{7} \approx 1.29$) then independent coalitions, i.e., $\{I_1, I_1, I_1, I_1, I_1\}$, are uniquely stable. Note that not all possible coalition structures can be stable. For example, the four-coalition structure $\{I_1, I_1, I_1, I_2\}$ is never stable.

Based on the above discussion, it is clear that the coalition formation decision depends on RASF in (4). The second component of RASF is a function of the risk level of the suppliers ($F$) and the size of the supplier base ($n$), while the type of supplier (complements or substitutes), and the form of the demand function determine the first component of RASF (i.e., $U(m)$). We discuss the detailed effects of the above four factors on the incentive to form alliances below.

Risk level of the suppliers ($F$): In the following corollary we generalize Theorem 1 and characterize the number of stable coalition structures (denoted by $m^*$) and the size of each stable coalition structure (denoted by $m_i$, where $1 \leq i \leq r$), in terms of the risk level $F$.

**Corollary 2.** The number of stable coalitions $m^*$ can only takes values in the set of integers $\{m_i\}_{i=1}^r$ satisfying $1 = m_1 < m_2 < ... < m_r \leq n$ where $r$ is an integer less than $n$. Furthermore, there exists a set of resource levels $\{F^{(i)}\}_{i=1}^r$ satisfying $0 < F^{(1)} < F^{(2)} < ... < F^{(r)}$ such that the grand coalition will be formed if $F \leq F^{(1)}$, $m_i$ coalitions will be formed if $F^{(i-1)} < F \leq F^{(i)}$ for $1 < i < r$, and $m_r$ coalitions will be formed if $F > F^{(r)}$.

Based on the above corollary (also refer to Example 2) it is evident that when the suppliers have significant amount of resources to reduce their default risks, they will prefer to operate alone or form alliances with small number of other suppliers (i.e., large $m^*$). On the other hand, if the suppliers themselves do not have access to much risk-mitigating resources, they would like to take advantage of resource-sharing by forming large alliances. Indeed, when the risk level is higher than a threshold level (equivalently, resource level is below $F^{(1)}$), a grand coalition is stable.

Size of the supplier base ($n$): Larger alliances are more achievable when the supplier base is smaller (lower $n$), i.e., when the intensity of direct competition among substitutable suppliers or indirect competition among complementary suppliers (Jiang and Wang 2010) is lower. This implies that in assembly systems, low-modularization design (that would involve small number of suppliers delivering these modules) facilitates cooperative decision making, whereas in substitutable case, cartels are more likely to be formed when there are less number of suppliers in the market. To illustrate this, consider Figure 4, which is based on Example 2. When $n = 3$, the grand coalition can be achieved for $F \leq F^{(1)} = 1.68$. When the number of suppliers increases to 10, however, grand coalition is only possible when the resource level is below the threshold $F^{(1)} = 0.17$.

The above two effects are summarized in Proposition 3.
PROPOSITION 3. In general, larger alliances are more likely to be formed
(i) among more risky suppliers (small $F$);
(ii) when the number of suppliers in the supply base is relatively small (small $n$).

As discussed before, the component/supplier type and the form of the demand function shape alliance incentive through their effects on $U(m)$. In order to understand this better, for the rest of this section we restrict our attention to the three most commonly assumed demand forms in the related literature — iso-elastic, linear-power (linear is a special case) and exponential (refer to Table A2 in the Appendix). Also, suppose that the pass-through rates, defined as the ratio of retail price change to the wholesale price change ($dp/dW$) (Tyagi 1999 and Moorthy 2005), of these demand functions are greater than 50%. Note that this assumption implies that the consumers will shoulder more of the change in wholesale prices than the retailer does, which is natural in many industries (Besanko et al. 2005).

Component/supplier type: It can be shown that $U(m)$ for the substitutable case is smaller than the complementary one implying that large coalitions are more achievable among complementary than substitutable suppliers. We illustrate this in the example below. This result is somewhat intuitive — since complementary suppliers are competing indirectly (rather than direct competition faced by substitutable ones), there is less reluctance on their part to enter into alliances.

Example 3. Consider iso-elastic demand $D(p) = ap^{-b}$ where $b = 6$. For complementary suppliers, $U^C(m) = \left(\frac{6-m}{5-m}\right)^5$. Theorem 1 shows that the grand coalition $\{I_5\}$ will be formed if $F \leq 1.05$; otherwise, independent coalitions $\{I_1, I_1, I_1, I_1, I_1\}$ will be formed. For substitutable suppliers,
\[ U^S(m) = \left( \frac{m+1}{m} \right)^2 \left( \frac{6 - 1/m}{6 - 1/(m+1)} \right)^3. \] If \( F \leq 0.59 \), then grand coalition will be formed; otherwise, independent coalitions \( \{I_i, I_j, I_k, I_l, I_m\} \) will be formed. Note that the condition for the grand-alliance is more restrictive for substitutable suppliers. □

**Structure of customer demand:** A further analysis of the demand functions also suggests that higher pass-through rate promotes the formation of larger alliances. Figure 5 illustrates this — as the pass-through rate increases, the upper-bound of the resource requirement in order for a grand coalition to be the stable equilibrium \( F_1 \) also increases. Recall that smaller alliances will most likely lead to higher wholesale prices and consequently higher retail prices (Proposition 1), and the pass-through rate measures the ratio of change in retail price over the change in wholesale price. Higher pass-through rate then implies a more sensitive vertical structure in which inefficient upstream decisions (e.g., small alliances, high wholesale prices) will lead to higher downstream retail prices. Thus, the jeopardy of small alliances is amplified as the pass-through rate increases, and hence large-alliance structure becomes more rewarding and stable. Based on Table A2 in the Appendix we can then conclude that iso-elastic demands with lower levels of elasticity or linear-power demands that more price-sensitive are more conducive to larger alliances.

The above two effects are summarized in the proposition below.

**Proposition 4.** In general, larger alliances are more likely to be formed

(i) among complementary than substitutable suppliers;

(ii) for supplier bases facing end customer demands with higher pass-through rates (Table A2).

![Figure 5](image-url)  
**Figure 5**  
Risk adjusted stability factor with index 1 \((RASF_1)\) varies with pass through rate
Risk-reduction vs competition-reduction: Until now we have focussed on characterizing the conditions under which alliances would be formed and whether it will be a large or a small one. A natural question that arises is whether alliance formation is driven by suppliers’ desire to reduce their risks or by the lure to reduce competition among themselves through cooperative decision-making. Indeed, we can answer this question by characterizing the stable coalition structure under a risk-less environment. Specifically:

Corollary 3. (CPNE in riskless environment) If the suppliers are effectively riskless (i.e., $F$ is sufficiently large), they will form the maximum number of possible coalitions ($m_r$) with the lowest possible sizes.

If the primary goal of alliance formation is competition reduction, suppliers should do so even when they have large amount of resources and are effectively riskless. But, the above corollary suggests that the least cooperative structure will arise when the resource level is above a threshold and there is no default risk. Note that this result conforms with Yin (2010), which studies a riskless environment for complementary suppliers. As regards to substitutable ones, we show that an independent structure is the stable equilibrium ($m_r = n$) for high values of $F$, as long as there are more than two suppliers in the supply base (refer to the Appendix)\textsuperscript{14}. So, clearly, the incentive for alliance formation lies not in competition reduction; rather, it is risk reduction through resource sharing that is conducive to alliance formation. Specifically, as $F$ increases reducing the suppliers’ risks, it also reduces the number of coalitions from $m_r$ down to $m^*$ and even to 1 (grand coalition), as discussed in Theorem 1.

5.3. Asymmetric Suppliers

In the previous section, we considered that each of $n$ suppliers has identical resource endowment $F$ and so are equally risky. In this section, we extend our analysis to the case of suppliers with asymmetric resource (i.e., risk) levels and study how the results of the previous section are affected. For the sake of analytical tractability, we assume that there are two possible resource endowment levels for the suppliers. Specifically, there are $n_L$ suppliers with low endowment level $L$ and $n_H = n - n_L$ suppliers with high endowment level $H(>L)$; i.e., there are $n_L$ high-risk and $n_H$ low-risk suppliers. Accordingly, there are two kinds of asymmetries in the supply base: 1) risk level asymmetry, which can be measured by the ratio $H/L$, and 2) demographic asymmetry, which can be measured by $n_H/n_L$. We first characterize the stable alliance structures that would prevail in the asymmetric case.

\textsuperscript{14}If $n = 2$, then $m_r = 1$ and a grand coalition will be stable.
Consider a coalition $S$ with $n_{SL}$ low-endowed suppliers and $n_{SH}$ high-endowed suppliers. It has a survival rate $G(F_S) = \frac{L_{n_{SL}} + H_{n_{SH}}}{1 + L_{n_{SL}} + H_{n_{SH}}}$, and suppose $S$ earns profit $\pi_S$ if it survives. Then the expected profit for a low-endowed supplier in this coalition is $\pi_S G(F_S) \left( \frac{L}{L_{n_{SL}} + H_{n_{SH}}} \right) = \frac{\pi_S L}{1 + L_{n_{SL}} + H_{n_{SH}}}$, and for a high-endowed supplier it is $\pi_S G(F_S) \left( \frac{H}{L_{n_{SL}} + H_{n_{SH}}} \right) = \frac{\pi_S H}{1 + L_{n_{SL}} + H_{n_{SH}}}$. We denote $F_{S_i}$ as the minimum resource level of suppliers in alliance $S_i$ and introduce the following definition.

**Definition 1.** *(m-similar-size-partition)* Given a set of suppliers $N$, the structures $\mathcal{S} = \{S_1, \ldots, S_m\}$ is an $m$-similar-size-partition of $N$ if it satisfies $\bigcup_{k=1}^{m} S_k = N$, $S_i \cap S_j = \emptyset$, and $F_{S_i} - F_{S_j} \leq F_{S_j}$ for all $i, j \in \{1, 2, \ldots, m\}$ and $i \neq j$.

The following theorem fully characterizes the stable coalition structures and demonstrates it to be quite similar to the one with identical suppliers, except that there is an adjustment in the $RASF_m$ expression of (4) to account for the asymmetry. Specifically, in this case, $RASF_m = U(m) - T_m$, where the $\{T_m\}_{m=1}^{n-1}$ depends on both risk and demographic profiles of the supplier base.

**Theorem 2.** *(CPNE among asymmetric suppliers)* If there are $n_L$ low-endowed suppliers with resource levels $L$ and $n_H$ high-endowed suppliers with resource levels $H$, where $n_L, n_H > 0$ and $H > L$, there exists a unique CPNE with $m^*$ coalitions. In particular, the suppliers

(i) will form a grand coalition ($m^* = 1$) if $U(1) \geq T_1$,

(ii) will act independently ($m^* = n$) if $U(n) \leq T_m$ for all $1 \leq m \leq n - 1$,

(iii) will form $m^*$ coalitions where $1 < m^* < n$, if $U(m) \leq T_m$ for all $1 \leq m \leq m^* - 1$ and $U(m^*) \geq T_{m^*}$, where $T_m = \min \{ \max \{ \frac{1 + F_{S_i}}{1 + F_{S_i}} \} : \mathcal{S} \text{ is an } m\text{-similar-size-partition of the supply base } N \}$ and the CPNE is the structure $\mathcal{S}$ that yields $T_{m^*}$.

As in the symmetric case, Theorem 2 also suggests an algorithm to find the stable coalitions by searching for the smallest $m$ at which $U(m)$ exceeds $T_m$, or equivalently, $RASF_m$ becomes non-negative. Unfortunately, $RASF_m$ is not straightforward to graph in the asymmetric case because of the difficulty in calculating $T_m$ with two endowment levels. Therefore, we provide an example to illustrate the application of Theorem 2.

**Example 4.** Consider the same problem parameters as in Example 2, i.e., $n = 5$ suppliers and linear demand $D(p) = 9 - 2p$, which yields $U(m) = \left(1 + \frac{1}{m+1}\right)^2$ for both complementary and substitutable suppliers. Suppose $L = 1/4, H = 5/6, n_L = 2$ and $n_H = 3$. There is only one $1$-partition of $N$, hence $T_1 = \frac{1 + F_N}{1 + L} = \frac{16}{5}$. By Definition 1, the set of $2$-similar-size-partitions of the supplier base contains only $\{H_2, L_2 H_1\}$, where $L_x H_y$ denotes a set of $x$ low-endowed suppliers and $y$ high-endowed suppliers. By using the expression for $T_m$ in Theorem 2, we can find $T_2 = \frac{28}{15}$.

Similarly, the set of $3$-similar-size-partition contains only $\{L_1 H_1, L_1 H_1, H_1\}$ and $T_3 = \frac{5}{3}$.
and the set of 4 – similar – size – partition contains only \{L_2, H_1, H_1, H_1\} and \(T_4 = \frac{6}{5}\). It can be verified that the smallest \(m\) at which \(U(m)\) exceeds \(T_m\) is 4. Therefore the CPNE contains 4 coalitions, and the partition of the suppliers in this collation structure is \{L_2, H_1, H_1, H_1\}. □

Based on the expression of \(RASF_m\) in the asymmetric case, it is evident that the effects of supplier type (substitutes or complements) and the customer demand function on the size of the alliances noted in Proposition 4 (for symmetric suppliers) remain valid since these two factors only affect \(U(m)\). In fact, even the effects of supplier risk levels and the size of the supplier base for identically endowed suppliers also carry over to the asymmetric case. Suppose \(\rho = n_H/n\) is the fraction of high-endowed suppliers in the community and \(F_N = n_L L + n_H H\) is the total of all supplier resources. Proposition 5 (i) characterizes a set of threshold conditions to identify stable coalition structures. This result implies that, in general, large alliances are more achievable when there are less high-endowed suppliers, or equivalently, when the risk level in the supply base is high (like Proposition 3 (i)). Proposition 5 (ii) analyzes the sensitivity of the thresholds with respect to the number of suppliers \(n\). The insight conforms with Proposition 3 (ii) in the sense that large alliances are more likely to occur when the supply base is relatively smaller.

**Proposition 5.** (i) There exists a set of \(\{\rho_i\}_{i=1}^{n-1}\) satisfying \(\rho_1 \leq \rho_2 \leq \ldots \leq \rho_{n-1}\) such that grand coalition will be formed if \(\rho \leq \rho_1\), independent coalition will be formed if \(\rho \geq \rho_{n-1}\), and \(m\) coalitions will be formed if \(\rho_{m-1} \leq \rho \leq \rho_m\) for any \(1 < m < n\).

(ii) \(\{\rho_i\}_{i=1}^{n-1}\) decreases with \(n\).

The main new insight we gain from this section pertains to the effects of the two asymmetries.

**Risk level asymmetry:** Recall that the degree of asymmetry between risk levels can be defined as \(H/L\). The following example illustrates how this ratio affect the alliance structure.

**Example 5.** In Example 4 above, \(H/L = 10/3\). Keeping the total endowment at the same level, i.e., \(F_N = 3\), we can reduce \(H/L\) by increasing \(L\) to \(1/2\), and decreasing \(H\) to \(2/3\). The ratio \(H/L\) becomes \(4/3\), which is now closer to 1. It follows that \(T_1 = \frac{16}{5}, T_2 = \frac{16}{9}, T_3 = \min\{\frac{13}{9}, \frac{7}{5}\} = \frac{7}{5}\), and \(T_4 = \frac{4}{3}\). Since \(U(1) = \frac{9}{4} < T_1\) and \(U(2) = \frac{16}{9} \geq T_2\), the CPNE now contains only 2 coalitions, and the stable structure can be identified as \{H_2, L_2H_1\}. □

The takeaway from the above example is that for large alliances to be stable, the resource levels of the supplier base cannot be too far apart from each other. This observation is formally established in Proposition 6.

**Proposition 6.** For a given demographic profile and total endowment level, larger alliances (small \(m^*\)) are more likely to be sustained by suppliers with similar risk levels (\(H/L\) closer to 1):
Demographic Asymmetry: This asymmetry comes from the number of suppliers in each group. Consider two sets of suppliers with the same total resource endowment $F_N$ and risk level asymmetry $H/L$, but the demographic distribution in one set is more asymmetric than the other, e.g., $n_H/n_L > n_H'/n_L' > 1$, then larger alliances are more likely to occur in the latter one.

Proposition 7. For a given risk profile and total endowment level, larger alliances are more likely to be formed when the supply base is more demographically similar ($n_H/n_L$ closer to 1).

Propositions 6 and 7 together suggest that homogeneity in risk and demographic profiles among suppliers incentivizes the formation of large coalitions, and vice versa.

6. Optimal Resource Investment Strategy

6.1. Upstream Suppliers

Until now we have assumed the risk-mitigating resource levels of the suppliers to be exogenous. However, investing in such resources can be expensive due to cost of capital (e.g., interest rate of the loans, loss of other initiative/investment opportunities) and has to be traded off against the benefits. In this section we address the issue of upstream suppliers making their resource investment decisions before the alliance formation decision in Figure 3 (i.e., Stage 0), thus endogenizing risk levels of the suppliers. Suppliers make these decisions competitively, and we characterize the resulting Nash equilibrium investment levels.

Suppose that a supplier’s cost of investing at level $F$ is $V_s(F) = vF$, where $v \geq 0$. We prove in the following that there is a unique resource level $F^I$ (symmetric equilibrium) that each supplier would invest in. The corresponding stable CPNE coalition structure can then be identified via Theorem 1, as $m^I = m^*(F^I)$.

Proposition 8. For $n$ suppliers each with investment cost rate $v$, there exists a unique $F^I > 0$ and $1 \leq m^I \leq n$ such that it is a Nash equilibrium for each supplier to invest at level $F^I$ and suppliers form $m^I$ coalitions of similar sizes. In particular,

$$F^I = \max\{F : \frac{v}{\pi(m^*)} \leq \frac{1+(n-1)F}{(1+nF)^2}\}, \ m^I = m^*(F^I),$$

where $m^*(\cdot)$ follows Theorem 1.

We illustrate the above result through the following example.

Example 6. Suppose that the supply base consists of five complementary suppliers ($n = 5$) whose end product has the market demand $D(p) = 9 - 2p$. With the aid of Table A2 and Proposition 1, we can plot $\frac{v}{\pi(m)}$ in Figure 6 as a step-wise function with respect to $F$ for different values of $v$.

\footnote{One interpretation of $v$ can be $v = (\text{commercial interest rate} + \text{annual depreciation rate}) \times \text{expected project term}.}$
In the first graph, we assume that the marginal production cost for each supplier is \( c = 0.04 \); hence the production cost of the end product is \( C = nc = 0.2 \). In the second graph, production costs are higher for both the parts (\( c = 0.4 \)) and the end product (\( C = 2 \)). Finally, the solid decreasing curves in both graphs represent \( \frac{1 + (n - 1)F}{(1 + nF)^2} \).

![Figure 6](image_url)  

**Figure 6** Equilibrium investment levels and coalition structure for Example 6

Now consider the example in which \( C = 0.2 \) and \( v = 5\% \). As shown in the first graph, the solid curve and the relevant step function cross at approximately \( F = 1.49 \) and \( m^*(F) = 5 \). Therefore, if the investment only costs 5% of the resource value, then each supplier would invest more, i.e., \( F^I = 1.49 \), and no alliance will be formed (\( m^I = 5 \)). On the other hand, consider the same production costs but higher investment cost, e.g., \( v = 20\% \). The equilibrium investment level would reduce to \( F^I = 0.77 \) and three coalitions (\( m^I = 3 \)) will be formed.

When production cost is high (\( C = 2 \)), there is no intersection for \( v = 20\% \). Yet \( F = 0.45 \) is the highest endowment level that makes the stair below the curve. By (5), \( F^I = 0.45 \) and \( m^I = 1 \). Thus, a grand coalition will occur. When \( v = 5\% \), \( F^I = 1.1 \) and \( m^I = 3 \).  

From Figure 6, the impact of production and investment costs on the equilibrium resource investments and coalition formations becomes evident. As the production cost increases, the margin becomes slimmer, making the loss due to supplier default more significant, and hence collaboration more attractive. The investment cost \( v \) also has a similar impact. An environment where getting credit is costly (high \( v \)) incentivizes suppliers towards more resource pooling, yielding lean investment levels and more cooperation in equilibrium. On the other hand, a lower cost of capital obstructs the formation of coalitions. Indeed, in the extreme case when the cost of capital is negligible, there is no incentive for the suppliers to form an alliance; this again supports our previous
assertion that risk reduction is the primary motive for alliance formation. The following corollary formalizes the above discussion.

**Corollary 4.** The number of stable coalitions $m^l$ decreases in investment cost $v$ and production cost ($C$ in assembly and $c$ in competitive markets).

Our analysis of the optimal resource strategy thus far has focused on symmetric equilibrium. In the appendix we have shown that asymmetric resource levels can also be stable under certain circumstances if investment is costly.

### 6.2. Downstream firm

Vertical resource investment/subsidy has been a proven practice between downstream firms and their suppliers (Babich 2010, Wadecki et al. 2012 and Wei et al. 2013). In our setting, although alliance decisions are mostly made by the suppliers themselves, there are examples of downstream firms like Unilever and Marks & Spencer trying to influence such decisions by providing proper financial incentives to their vertical partners (Oxfam International 2010). In this section, we briefly discuss the case where the investment decision about risk-mitigating resources is made by the downstream party and analyze the role the downstream firm (the assembler or the buyer) can play in encouraging (or discouraging) the suppliers to form alliances.

Like in §5, we analyze a case where there are $n_L$ suppliers with resource levels $L$ and $n_H = n - n_L$ suppliers with $H$. Since the suppliers’ incentives for alliance formation can be indirectly influenced by changing the degree of asymmetry among the suppliers, we restrict our attention to the following three strategies for the downstream party:

- **Floor Lifter (FL):** upgrades resource levels of all $L$ suppliers to $L' > L$;
- **Roof Lifter (RL):** upgrades resource levels of all $H$ suppliers to $H' > H$;
- **Elite Coupon (EC):** upgrades resource levels of some $L$ suppliers to $H$.

Clearly, the first strategy reduces the *risk level asymmetry* among the supply base, while the second strategy increases it. On the other hand, the third strategy affects *demographic asymmetry*. Assume that for the downstream firm the cost of upgrading a supplier’s resource level from $F$ to $F + \Delta F$ costs $V_d(\Delta F) = u + v\Delta F$. Here $u$ represents the fixed cost in dealing with one supplier, and $v$ the investment cost rate. Therefore, $u$ and $v$ capture the costs in altering the *demographic distribution* and *risk level*, respectively. We next apply the above three strategies to complementary/substitutable suppliers and study which one is preferred by the assembler/buyer.

**Proposition 9.** In general,

- an assembler facing complementary suppliers should adopt (FL) or do nothing,
- a buyer facing substitutable suppliers should adopt (RL) if $u$ and $v$ are small, or if there are few high-endowed suppliers ($n_H$ small); otherwise it should adopt (EC) or do nothing.
From Corollary 1 in §4 we know that an assembler would have an incentive to build larger coalitions and so she would always focus on making the suppliers more homogenous. Her optimal strategy is then to invest in increasing the resource levels of all low-endowed suppliers so that the resources of the two supplier groups are more similar. So, her investment strategy has a large scope (all suppliers), but limited depth (only increasing from $L$ to $L'$).

The buyer, on the other hand, would want to break a stable grand coalition. So, it is possible for the buyer to go for a depth-oriented strategy, EC, where she invests in a small number of low-endowed suppliers to reduce their risks. The suppliers can become more asymmetric demographically ($n_H/n_L$ increases) and a small set of low-endowed suppliers may be left out to fend on their own. So, clearly, the nature of the components plays a crucial role in determining the scope and depth of the resource investment of the downstream firm.

7. Robustness Analysis

To focus on the core issues related to supplier alliances, we made several assumptions in our main analysis. In this section, we briefly discuss the implications of relaxing some of these assumptions and identify to what extent the insights and results are affected.

7.1. Allocation Rules

Our results so far are based on the proportional allocation rule (2). Note that the existing cooperative game literature in supply chain management area mostly uses allocations based on Shapley values (i.e., equal allocation in absence of default risk; see Nagarajan and Sošić 2007, Granot and Yin 2008). In our problem, it turns out that the Shapley allocations are indeed equivalent to the proportional rule when suppliers are symmetric, or when risk levels are high (equivalently, resource levels are low) and the survival probability $G(F)$ can be approximated by $G(F) = \frac{F}{1 + F} \approx F$.

Now consider a generalization of our allocation scheme, in which $\gamma_{S_k} = \frac{F^u}{\sum_{j \in S_k} F_j^u}$ for some $u \geq 0$. This family of allocation rules are quite general. In particular, through such mechanism, the profit can be allocated proportionally ($u = 1$ as the main analysis assumes), in favor of high-resource-share suppliers (when $u > 1$) or low-resource-share suppliers (when $0 < u < 1$).

Based on the generalized allocation rule, if there are two groups of asymmetric suppliers like in §5.3, the expected profit for a low-resource supplier in a survived alliance $S_k$ is $\pi_S G(F_S) \left( \frac{L^u}{L^u n_{SL} + H^u n_{SH}} \right) = \pi_S \left( \frac{L_{n_{SL}} + H_{n_{SH}}}{1 + L_{n_{SL}} + H_{n_{SH}}} \right) \left( \frac{L^u}{L^u n_{SL} + H^u n_{SH}} \right)$, and that for each high-resource supplier is $\pi_S \left( \frac{L_{n_{SL}} + H_{n_{SH}}}{1 + L_{n_{SL}} + H_{n_{SH}}} \right) \left( \frac{L^u}{H^u n_{SL} + H^u n_{SH}} \right)$. We can then characterize the condition for a grand coalition to be stable as shown in the following theorem.

**Theorem 3.** (Grand coalition among asymmetric suppliers under general allocations) For a set of $n_L$ suppliers with resource levels $L$ and $n_H$ suppliers with resource levels $H$, do it...
where \( n_L + n_H = n \), \( H > L \) and \( F_N = n_L L + n_H H \), a grand coalition is a CPNE if and only if
\[
U(1) \geq \max \left\{ \frac{L^{1-u}}{1+L}, \frac{H^{1-u}}{1+H} \right\} \frac{(n_L L^u + n_H H^u)(1 + F_N)}{F_N}.
\]

Clearly, the grand coalition characterization is similar to that for proportional allocation (Theorem 2) with adjustments to account for varying \( u \). Furthermore, for a high risk environment (small \( L \) and \( H \)), we are able to elaborate more on the impact of allocation rules. Overall, we find that the CPNE structure and conditions are similar to our main results in §5.3. The most interesting new insight is that large alliances can be induced by “fair” proportional allocation (\( u = 1 \)), as in the rest of the paper; moreover, even when the allocation rule is not proportional (i.e., \( u \neq 1 \)), large alliances can be stable provided that suitable values of \( u \) make allocations relatively fairer via favoring the minority in the supplier community. For example, when \( n_L < n_H \), an “egalitarian” allocation with an appropriate \( u < 1 \) (which assigns dis-proportionately high allocation to the low-endowed suppliers) is more likely to induce the grand coalition than an “elitist” allocation with \( u > 1 \) (assigning dis-proportionately high allocation to the high-endowed suppliers). The reverse holds when \( n_L > n_H \).

### 7.2. Default Premium on Wholesale Prices

In analyzing the key trade-off of joining small versus large alliances, we assumed that a defaulted supplier can be replaced in a frictionless manner (i.e., \( \delta = 0 \)). While such an assumption maybe realistic under certain scenarios, in this section we allow \( \delta > 0 \) and investigate how it may affect alliance structures. That is, as discussed before, the downstream firm will take into account the possibility that each of the alliances might default with certain probability, in which case she would have to pay a premium \( \delta S_k \) to ensure supply, where \( \delta S_k = |S_k| \delta \) for complementary components and \( \delta S_k = \delta \) for substitutable ones.

In the appendix we show that for symmetric suppliers the alliance structure in §5.2 remains valid as long as the premium \( \delta \) is not too large. Specifically, we can use a set of refined risk adjusted stability factors (RASFs), which now depend also on \( \delta \), to identify the stable coalition structures. Rather than going into the details, we illustrate it with the following example.

**Example 7.** Consider the same scenario as in Example 2, i.e., an assembly system with \( D(p) = 9 - 2p \), \( c = 0.2 \), and \( n = 5 \). According to Lemma A5 in the Appendix, the structure in Theorem 1 holds as long as \( \delta \leq \frac{9/2 - 1}{2 + 5} = 0.5 \). Now assume that \( \delta = 0.1 \). Based on Figure 7, we can then deduce if the endowment level \( F \leq 0.5 \), then a grand coalition is uniquely stable. If \( 0.5 < F \leq 0.72 \), then a two coalition structure \( \{I_2, I_3\} \) is stable. When \( 0.72 < F \leq 1.5 \), the coalition structure \( \{I_1, I_2, I_2\} \) is uniquely stable. For \( F > 1.5 \), all RASF’s are negative, which leads to independent coalitions as the only CPNE. Four-coalition structure is never stable. \( \square \)
Figure 7  Risk adjusted stability factor among complementary suppliers with $D = 9 - 2p$, $n = 5$, $c = 0.2$ and $\delta = 0.1$

Figure 8  $RASF_1$ for complementary suppliers with variation in (i) pass through rates, (ii) total production costs, (iii) default premium/component cost ratios

The coalition structures in the above example with $\delta > 0$ are quite similar to Example 2 with $\delta = 0$, except for the threshold endowment levels. The threshold $F$ values are higher for $\delta > 0$, implying that suppliers will be more incentivized to join larger alliances and less coalitions should be observed in equilibrium for a positive default premium. Our numerics suggest this insight to be quite robust. They also suggest that the curvature of $RASF$ with $\delta > 0$ is robust to problem parameters. Figure 8 shows the change of $RASF_1$ with respect to pass-through rate (Table A2), total production cost ($C = nc$) and the default premium to component cost ratio $\delta/c$. In general, the curves shift rightwards as these factors become more significant — indicating that stable big alliances are more achievable when the downstream is more sensitive to upstream price changes,
when the raw material/labor is more costly, and when the default penalty is more severe. Note that the threshold $\delta$ for our results to be valid is not too restrictive. In Example 7, the maximum $\delta$ is 50% of the total production cost $cn = 1$. In the Appendix, we show that for linear demand, the threshold $\delta$ approximates the maximum margin each stakeholder can earn.

Lastly, we comment that we assume a per unit premium $\delta$ in our analysis. An alternative assumption is a lump sum payment, reflecting the fixed searching/expediting fees of emergency sourcing. That is, if a coalition $S_k$ defaults, the assembler/buyer can still procure at the pre-announced wholesale price $w_{S_k}$, yet it costs her $\Delta_{S_k}$ to seek new suppliers. All results in §4-§6 hold unconditionally for this alternative assumption.

8. Concluding Remarks
Cooperative alliances with other firms are important levers that an organization may seek beyond its internal measures. An alliance equips its members with better resources and opportunities, and also changes the way they operate and envision themselves. This phenomenon is observed in a number of industries including agricultural, pharmaceutical and manufacturing. Alliances are especially effective in dealing with external risks that a firm might be facing. However, given the individual and collective profit motives that an alliance must satisfy, the incentives to form alliances and their stability are issues of considerable research interest. So far, supply chain management literature has studied alliances focusing primarily on demand-side risks. However, one of the most salient features of the recent business environment has been a significant increase in the supply-side risk, especially in supplier default. The objective of this paper is to understand what types of stable supplier alliances will develop in the presence of the risk of default, and how the suppliers’ alliance formation incentives are shaped by the business environment.

In order to achieve our objective, we use a channel framework consisting of $n$ upstream suppliers and one downstream firm where the suppliers face default risk and can form alliances to counteract such risk. Our framework is applicable for both complementary and substitutable suppliers and has a number of other characteristics that distinguishes it from the existing literature. Specifically, each supplier has access to a certain amount of generic, shareable resources to mitigate his default risk and entering into alliances can further reduce such risk through resource sharing among partners, although the risk-mitigating benefit of resources exhibits diminishing returns. Also, the profit allocation mechanism among the partners in an alliance is proportional to their resource shares (equivalent to Shapley value based allocation when risk levels are relatively high). The above enables us to deal with an important trade-off a supplier faces while deciding on whether to join an alliance not captured before in the literature — doing so decreases a supplier’s default risk but also might have adverse implications in terms of his profit share.
We first focus on the scenario where all suppliers are symmetric in terms of the amount of exogenous risk-mitigating resources they possess and fully characterize the stable alliances that will develop among them in equilibrium. It turns out that the sizes and number of stable alliances depend primarily on a measure, termed Risk Adjusted Stability Factor (RASF), that succinctly captures the business environment of the suppliers. Further analysis of this measure suggests that larger alliances are stable when: i) the suppliers are more risky (i.e., their resource levels are lower), ii) the supplier base is smaller, iii) the suppliers are complements (rather than substitutes), and iv) the pass through rate of the customer demand is higher. On the other hand, the converse business conditions result in smaller alliances. One of the most important insights of this analysis is that suppliers form alliances in order to reduce risks through resource sharing, rather than reduce competition. Consequently, if suppliers are risk-free, they do not form any alliances (i.e., act independently), and when they are very risky, they form a grand coalition. We also characterize the exact composition of stable alliances that will develop even when the suppliers are asymmetric in terms of their resources (risks). All the above insights continue to hold, except that as the asymmetry — either in terms of the resource levels or demography — in the supplier base increases, it becomes more difficult to sustain large alliances.

Traditionally, stability of grand coalition has been important to anti-trust authorities because of its implications for monopoly power\textsuperscript{16}. However, our context brings another aspect of such alliances to light because they are least prone to default. So, conditions for stable large (specifically, grand) alliances result in less risky supply chains. Anti-trust authorities need to keep this positive impact in mind while evaluating them. Moreover, our analysis also suggests certain rationale as to why we see more alliances in industries like automobile (Geneva 2005) and agriculture (Oxfam International 2010). The former may be attributed to the complementary nature of the components while the latter may come from the fact that most of the members of organizations like FPOs in India are small farmers who face significant amount of risks and are relatively similar in terms of their resource endowments.

We then consider the case where the suppliers first decide on their resource investment levels before their alliance formation strategy. In this case, we deduce the optimal levels of investments that each supplier will undertake in order to reduce their risks and keeping in mind the alliance formation opportunities. It turns out that if the investment cost is relatively low (e.g., in the present interest rate environment), they will invest significantly and not form alliances so as not

\textsuperscript{16} However, in the United States, agricultural cooperatives are generally protected by the Capper–Volstead Act from antitrust laws. In Canada, strategic alliances are empowered with joint pricing/operational decision making and exempted from pro-competition laws, if the alliances are under a broader guideline that generates efficiencies or innovation (Government of Canada 2002).
to share the profits. On the other hand, if resource investment is costly, each of them will not invest much and depend on resource sharing in large alliances to reduce their risks. The findings on horizontal alliance formation also generates prescription for downstream supply chain parties to influence upstream coalition structures through appropriate investments.

Lastly, we remark that the results of this paper are quite robust. For example, they hold true irrespective of whether or not a default premium is to be paid in addition to the wholesale price in case of a supplier default, as long as the premium is not too high. The primary effect of a positive default premium is that it increases the sizes of the alliances and reduces their number. Similarly, generalizations in terms of non-proportional profit allocations also support our results. However, if the profit allocation is not proportional (i.e., fair), then larger alliances can only be sustained if the mechanism provides disproportionate favoritism towards minority supplier types.

While this paper tackles how default risk would affect alliance formation and how suppliers trade off the pros and cons of such a strategy, there are certainly many ways to extend this line of work. A more in-depth study would call for further differentiation among the suppliers – going further than the two risk levels, with more refined characterization of the type and magnitude of the risks they are exposed to. Extending the analysis from the current channel structure to general supply chain networks would also be interesting. One could also consider other types of supply-side risks, including random yield and fluctuating raw material costs, possibly bringing risk correlation into the picture. Another possible avenue for future research is to focus on resources that are product- or relation-specific, e.g., inventory or capacity, and hence can only be shared under certain circumstances. This will possibly require separate analysis for complementary and substitutable cases. Empirically testing the propositions to identify missing factors and industry-based coefficients would be another potential line of work.

**References**


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